

Is $Q=CiA$ Dimensionally Consistent?

Shed runoff flow rate Q has been computed under the Rational Method using the equation:

$$Q = CiA$$

for a very long time, but ever so often, a new user would ask if this equation is dimensionally consistent.

In this equation, the units of Q is cfs (cubic feet per second, or ft^3/sec). C is the runoff coefficient, which is dimensionless, i is the rainfall intensity in inches per hour (in/hr), and A is the tributary watershed area, in acres. If the equation were dimensionally consistent, then shouldn't the units of Q be ($\text{in} \cdot \text{ac}/\text{hr}$)? If it is, then is $\text{in} \cdot \text{ac}/\text{hr} = \text{ft}^3/\text{sec}$? This is examined below.

Convert $\text{in} \cdot \text{ac}/\text{hr}$ to ft^3/sec as follows:

$\text{in} \cdot \text{ac}/\text{hr}$	$= (1/3600) \cdot \text{in} \cdot \text{ac}/\text{sec}$
	$= (1/12) \cdot (1/3600) \cdot \text{ft} \cdot \text{ac}/\text{sec}$
	$=$ $(1/12) \cdot (1/3600) \cdot 43560 \cdot \text{ft} \cdot \text{ft}^2/\text{sec}$
	$= (43560 / (12 \cdot 3600)) \text{ft}^3/\text{sec}$
	$= 1.0083 \cdot \text{ft}^3/\text{sec}.$

Thus, strictly speaking, $Q = 1.0083 \cdot CiA$ for English units, which is usually rounded to $Q = CiA$. Thus, $Q = CiA$ is indeed dimensionally consistent. The Rational Method equation, however, slightly underestimates Q , by 0.83 percent, for English units.

Similarly, it may be shown that the metric equation:

$$Q = (1/360) \cdot CiA,$$

where Q is the watershed runoff flow rate with units of m^3/sec , C is the dimensionless runoff coefficient, i is the rainfall rate in mm/hr , and A is the tributary watershed area in hectares, is dimensionally consistent, without the need for rounding. The Rational Method equation does not underestimate (or overestimate) Q for metric units.